HMMT Acceptable Answer Forms

Head Graders, HMMT 2012

1 General Policies

When grading tests at HMMT, we aim to ensure that students are awarded credit for any problems they've solved completely and correctly. In particular, this means that we don't want to be pedantic about the forms in which answers are expressed−as long as we can quickly identify an answer as unambiguously correct, we'll award it points.

You are always allowed to express answers using any basic arithmetic operations ($+, -, \times, /$), parentheses, exponents, factorials, trigonometric functions, inverse trigonometric functions, and binomial coefficients, as well as any other notations allowed explicitly by a problem. You generally are not allowed to use summation or product notation (i.e. Σ and Π notation) to express answers. Correct mathematical notation must always be used.

If a problem calls for a list (e.g. "Find all solutions $x \ldots$"), you will be awarded full credit for a complete list in any order, but no partial credit for an incomplete list, as well as no credit for a list with extra elements. Partial credit is awarded only on the Team Round.

2 Guidelines for Acceptable Answers

When grading, we compare each student's answer to a problem with our answer key for that problem. If a student's answer is equivalent to one of the answers in our key, we want to award points−however, we don't always have the resources to determine whether a complicated or otherwise strange answer is equivalent to ours. Further, an answer may be technically correct, but left so unsimplified that an important part of the solution has been skipped. Therefore, to guarantee that your correct work is rewarded, your answers should be as *simplified* as possible.

For your answer to be *simplified*, you should:

(1) carry out any reasonable calculations. For instance, you should evaluate any expressions which will take negligible time to evaluate (such as $\frac{1}{2} + \frac{1}{3}$ and 2×10^5), as well as all products less than 10^4. Unreasonable calculations include large powers (e.g. 3^{20}), large factorials, products which are greater than 10^4, and trigonometric functions which cannot be expressed in terms of radicals.

(2) write rational numbers in lowest terms. Decimals are also acceptable, provided they are *exact*. $\frac{1}{7}$ may be written as $.\overline{142857}$, but not, for example, as $.142857$. Unless

specified otherwise, irrational numbers may never be approximated as rational (e.g. 3.14 and $\frac{22}{7}$ are not acceptable in place of π).

(3) move all square factors outside radicals–for instance, write $3\sqrt{7}$ instead of $\sqrt{63}$. However, denominators need not be rationalized. Both $\frac{2\sqrt{5}}{5}$ and $\frac{2}{\sqrt{5}}$ are acceptable. Radicands are also allowed to contain nested radicals.

Here are some examples of simplified answers, and also some unsimplified ones with their simplified alternatives:

SIMPLIFIED
$\binom{23}{6}$
7^5
$3 + 3 \times 2^{16}, 3(1 + 2^{16})$
$10!$
$\frac{1}{3} - 2^{-16}, \frac{2^{16}-3}{3 \times 2^{16}}$
$\sqrt[4]{\frac{5}{4}}$
$\cos 1$
$\frac{1}{1+\sqrt{2}}, \sqrt{2} - 1$
$\frac{10}{\sqrt{5}}, 2\sqrt{5}$
$\sqrt{6 + 2\sqrt{5}}, 1 + \sqrt{5}$

UNSIMPLIFIED	SIMPLIFIED
24×14	336
$\frac{6}{4}$	$\frac{3}{2}, 1.5$
$\frac{2^{50}-2^{40}}{12}$	$\frac{2^{48}-2^{38}}{3}$
7^4	2401
$7!$	5040
$\binom{30}{3}$	4060
$\sqrt{\frac{5}{4}}$	$\frac{\sqrt{5}}{2}$
$\sqrt{6.75}$	$\frac{3\sqrt{3}}{2}$
$\sin\left(2\arcsin\left(\frac{5}{13}\right)\right)$	$\frac{120}{169}$
$\cos(\frac{\pi}{3})$	$\frac{1}{2}$

Finally, remember that these guidelines serve not to give us the grounds not to award points for correct answers, but rather to speed up grading by having a smaller set of acceptable answers for each problem. If an answer does not strictly follow these guidelines we may still award it points, at the discretion of the Head Graders−in this case, we will award points for this alternative answer completely consistently. Any correct answer following these guidelines will be awarded points.

Questions may be referred to hmmt-request@mit.edu.